

Book Review

D. Goeleven, “*Noncoercive variational problems and related results*”, Pitman Research Notes in Mathematical Sciences 357, Addison Wesley Longman (1996), ISBN 0582 304024.

In recent years, many engineers and mathematicians have directed their attention towards noncoercive problems. The reason is that various engineering models lead, generally, to a noncoercive problem. The lack of coercivity is due to the fact that, mainly because of the boundary conditions, the kernels of some bilinear forms of the problem are nonempty linear subspaces. This is the case, e.g. in classical elasticity, if the boundary conditions permit rigid body displacements. Analogous is the case of buckling problem.

The book deals with the study of variational problems which can be formulated as the minimization of some convex or nonconvex, and noncoercive functionals. Some additional results and possible prospects concerning the field of nonvariational problems are also considered.

The main object of the work is to present a general mathematical existence theory applicable to the study of noncoercive mathematical models arising in Nonsmooth Mechanics. Particularly, the author directs his efforts to the study of problems which can be formulated by means of variational inequalities and hemivariational inequalities. However, other approaches and problems are considered too since the scope of this work is also to unify or generalize some known results and to connect several theories.

The author presents an approach relying on the asymptotic behavior of the sets, functions and operators which are involved in a considered problem. This approach is called the recession approach and it appears through this work as a particularly useful approach to deal with semicoercive unilateral problems and several types of nonlinear perturbations of semicoercive unilateral problems.

Definitions and general properties of the recession functions, weak-coercivness conditions and other preliminaries are given in Section 2. In Section 3.1, the author considers an abstract minimization problem in a general framework and combines a Tychonov regularization with a general recession approach in order to present a constructive theory. In Section 3.2, the results of Section 3.1 are reviewed when one replaces the Tychonov regularization by a general viscosity approach. Section 3.3 concerns the extension of the recession analysis for the study of nonvariational problems invoking possibly nonconvex sets of constraints. A

general class of hemivariational inequalities is there investigated. Section 4 deals with the applications of the abstract recession method. One shows that a unique methodology can be used in order to study many different problems. In Section 4.1, one considers an elliptic problem with unilateral boundary conditions. Section 4.2 deals with the study of a model for the description of a masonry-like problem. In Section 4.3, D. Goeleven discusses some basic global minimization theorems. In Section 4.4, one refines the recession analysis when a separability property holds for the generally nonsmooth 'energy' functional. Section 4.5 deals with the study of noncoercive unilateral problems arising in nonlinear elasticity and involving locking constraints. In Section 4.6, one shows how weak convergence methods can be used in order to check a compactness condition used in the recession analysis. Section 4.7 concerns the minimization of noncoercive functionals on manifold type constraints. Section 4.8 deals with the study of some nonlinear and noncoercive partial differential equations. Finally in Section 4.9, 4.10, 4.11 and 4.12, the author uses the recession analysis in order to prove new results in the theory of variational inequalities and hemivariational inequalities, and with Section 4.13 ends the book with the presentation of a linear ill-posed system.

In conclusion, the book presents in a rigorous and clear manner a general theory for the existence of solutions of noncoercive variational problems and of constrained problems formulated as variational inequalities and hemivariational inequalities. The book illustrates very successfully recent mathematical approaches and results with various examples from mathematics and mechanics. Moreover it unifies ideas for the treatment of various noncoercive problems and contains unpublished original results for variational inequalities and hemivariational inequalities.

The book is written with clarity and originality and is addressed to the applied mathematician and to the mathematically oriented engineers.

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